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ELEMENTARY THEORY OF PSEUDO-TURBULENCE IN FINELY-DISPERSED SUSPENSIONS

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UDC 532.545

Relations are obtained to characterize the standard deviations of fluctuation velocity and the self-diffusion coefficients of the phase in flows of suspensions of small particles.

Suspended particles and fluid moles are brought into small-scale pulsative (pseudo-turbulent) motion even in flows of suspenions which are macroscopically uniform, and this motion has a significant effect on the distribution of the phases in the flows and the effective heat- and mass-transfer coefficients. Energy for the pulsations is supplied by the work done by the carrier flow against fluctuations in the concentration of the suspension. The forces acting on individual particles differ from the local mean value, which leads to acceleration of the particles. As they accelerate, the particles entrain adjacent moles of fluid [1, 2].

The theory of pseudo-turbulent motion is based on representation of the fluctuations of the concentration, pressure, and velocity of the particles and the fluid in the form of steady-state random functions. These functions are analyzed using the equations of fluctuational gasdynamics, which are in turn obtained directly from the averaged equations of mass and momentum conservation for the phases of a suspension [3, 4]. The calculations prove to be very cumbersome in this case. In addition, there is a logical contradiction; the linear scale of the unknowns in the averaged equations is assumed to be much greater than the dimensions of the particles in the suspension, but these equations are in essence being used to describe fluctuations with a scale on the order of these dimensions. Here, we attempt to circumvent this problem and at the same time simplify the calculations.

We will examine a flow of a monodisperse suspension of fine spherical particles. The local values of the mean phase velocities and the pressure and concentration of the particles are determined from the solution of the hydrodynamic equations of the suspension and, in the analysis of pseudo-turbulence, are assumed to be known quantities independent of the coordinates and time. The latter assumption is justified by the fact that the temporal and linear scales of these means must be significantly greater than the corresponding scales for the pulsations.

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The Lagrange equation for a particle is obtained by taking the equation expressing Newton's second law and subtracting the corresponding averaged equation. The Lagrange equation can be written in the form $md\mathbf{w}'/dt = \mathbf{F}'$. Multiplying this by the mean numerical concentration of particles n — which is assumed to be constant — we obtain the fluctuation equation

$$\rho d_1 d\mathbf{w}'/dt = n\mathbf{F}'. \tag{1}$$

In the case of sufficiently fine particles, the phase interaction force is equal to the sum of the Stokes, Basse, and Faxen forces, buoyancy, and the forces associated with acceleration of the apparent additional mass of the fluid. This sum was calculated in [5] for a suspension of moderate concentration. A component of the phase interaction force which is dependent on the gradient of the mean concentration may also be manifest in a macroscopically nonuniform suspension. Evaluating these forces is difficult in a highly concentrated suspension. However, in keeping with obvious physical considerations, it can be assumed that the main contribution to the fluctuation force $\mathbf{F}'$ will be made by the component due to viscous interaction of the particles and the fluid. This component in turn depends on the local concentration of the disperse phase in the neighborhood of the given particle. The analogous component of the total force acting on the particle can be represented as $6\pi a\mu_0 K.(\mathbf{v}-\mathbf{w})$, where K is a function of ρ which increases monotonically from unity at $\rho = 0$. Thus, we approximately take $n\mathbf{F}'/\rho \approx \beta K(\mathbf{v}'-\mathbf{w}') + \beta(dK/d\rho)\mathbf{u}\rho'$, where $\beta = 9\mu_0/2a^2$; $\mathbf{u} = \langle\mathbf{v}\rangle - \langle\mathbf{w}\rangle$, while K and its derivative are calculated with $\rho = \langle\rho\rangle$ (to simplify the notation, we will henceforth omit the brackets for the means). If $d_0 \sim d_1$, then ignoring the inertial terms in the right side of (1) also requires ignoring the inertia of the particle itself. Otherwise, the error will be too large. Thus, in the noninertial approximation being examined, we have

$$\mathbf{v}' - \mathbf{w}' + (d\ln K/d\rho)\ \mathbf{u}\rho' = 0. \tag{2}$$

It should be noted that we also ignored the resulting effect of direct collisions between particles in (2). This omission corresponds to a collision-less approximation in which all interactions between particles are mediated by the fluid.

Ignoring the fluctuation of buoyancy in (2) corresponds to the assumption that the particle is suspended in a medium with a constant density $d = \varepsilon d_0 + \rho d_1$. This is in turn equivalent to the assumption $\rho = \text{const}$, which we will use in analyzing the fluctuations of the velocity of the fluid moles.

The analog of Eq. (1) for the fluid has the form

$$\varepsilon d_0 \partial\mathbf{v}'/\partial t = -\nabla p' - n\mathbf{F}'. \tag{3}$$

For simplicity, here we ignored the viscosity of the fluid. Also, examining motion in the coordinate system connected with the mean local velocity of the suspension and assuming u to be small, we replaced the complete derivative with respect to time by the partial derivative; we can proceed similarly with (1). Adding (1) and (3), we arrive at the equation

$$\varepsilon d_0 \frac{\partial\mathbf{v}'}{\partial t} + \rho d_1 \frac{\partial\mathbf{w}'}{\partial t} = -\nabla p', \tag{4}$$

which we augment by the corresponding continuity equation: $\nabla\mathbf{v}' = 0$.

We then follow [3, 4] and use the theory of steady-state random processes [6], representing the random variables φ' and ψ' in the form of Fourier–Stieltjes integrals:

$$\{\varphi',\ \psi'\} = \int \exp[i(\omega t + \mathbf{kx})]\ \{dZ_\varphi,\ dZ_\psi\},$$

where dZ_φ, dZ_ψ are the corresponding random measures. The double-point, double-time correlation function of these quantities is then represented in the form [6]

$$\langle \varphi'(t,\ \mathbf{x})\psi'(t+\tau,\ \mathbf{x}+\mathbf{r})\rangle = \int\int \exp[-i(\omega\tau + \mathbf{kr})]\ \Psi_{\varphi,\psi}(\omega,\ \mathbf{k})\, d\omega d\mathbf{k},$$

$$\Psi_{\varphi,\psi}(\omega,\ \mathbf{k}) = \langle dZ_\varphi dZ_\psi^* \rangle / d\omega d\mathbf{k},$$

Meanwhile, with $\tau = 0$ and $\mathbf{r} = 0$, we obtain an expression for $\langle\varphi'\psi\rangle$.

The spectral density of the fluctuations of volumetric concentration of particles will be approximated by the expression [7]

$$\Psi_{\rho,\rho}(\omega,\ \mathbf{k}) = \frac{\langle dZ_\rho dZ_\rho^* \rangle}{d\omega d\mathbf{k}} = \frac{\mathbf{kDk}}{\pi} \frac{\Phi_{\rho,\rho}(k)}{\omega^2 + (\mathbf{kDk})^2},$$

$$\Phi_{\rho,\rho}(\mathbf{k}) = \begin{cases} \Phi, & k < k_0, \\ 0, & k > k_0, \end{cases} \quad \Phi = \frac{3}{4\pi} \frac{\rho^2}{k_0^3} \left(1 - \frac{\rho}{\rho_*}\right), \tag{5}$$

$$k_0 = (9\pi\rho/2)^{1/3} a^{-1},$$

where ρ_* is the maximum permissible volumetric concentration of particles in the flow corresponding to their packing density. Equations (5) constitute a simplified model in which the disappearance of fine-scale fluctuations of concentration is described by means of a parabolic effective-diffusion equation with the tensor of the self-diffusion coefficients of the suspended particles **D**.

We can write the following for the components of the tensors of the self-diffusion coefficients of the particles and fluid moles in the approximation being examined [3, 4]

$$D_{ij} = \pi \int \Psi_{wi,wj}(0,\ \mathbf{k})\, d\mathbf{k}, \quad E_{ij} = \pi \int \Psi_{vi,vj}(0,\ \mathbf{k})\, d\mathbf{k}.$$

In these formulas, integration is carried out over the entire axis of frequencies ω and wave space $\mathbf{k}$. The first problem is to find representations for the different spectral densities $\Psi_{\varphi,\psi}(\omega, \mathbf{k})$ through the known spectral density (5) of the volumetric concentration fluctuation.

Equations (2) and (4) and the continuity equation lead us to a system of linear algebraic relations for the random measures:

$$d\mathbf{Z}_v - d\mathbf{Z}_w = (d \ln K/d\rho)\, \mathbf{u} dZ_\rho,$$

$$\omega(\varepsilon d_0 d\mathbf{Z}_v + \rho d_1 d\mathbf{Z}_w) = -\mathbf{k} dZ_p,\ \mathbf{k} d\mathbf{Z}_v = 0,$$

which in particular leads to

$$d\mathbf{Z}_v = -\frac{\rho d_1}{d} \frac{d \ln K}{d\rho} \left[\mathbf{u} - \frac{(\mathbf{uk})\,\mathbf{k}}{k^2}\right] dZ_\rho,$$

$$d\mathbf{Z}_w = \left[\left(1 - \frac{\rho d_1}{d}\right)\mathbf{u} + \frac{\rho d_1}{d} \frac{(\mathbf{uk})\,\mathbf{k}}{k^2}\right] \frac{d \ln K}{d\rho} dZ_\rho, \tag{6}$$

$$d = \varepsilon d_0 + \rho d_1$$

and then

$$\Psi_{vi,vj} = \left(\frac{\rho d_1}{d}\right)^2 \left[u_i u_j - \frac{(u_m k_m)(u_i k_j + u_j k_i)}{k^2} + \frac{(u_m k_m)^2 k_i k_j}{k^4}\right] \left(\frac{d \ln K}{d\rho}\right)^2 \Psi_{\rho,\rho},$$

$$\Psi_{wi,wj} = \left[\left(1 - \frac{d_1\rho}{d}\right)^2 u_i u_j + \frac{d_1\rho}{d}\left(1 - \frac{d_1\rho}{d}\right) \frac{(u_m k_m)(u_i k_j + u_j k_i)}{k^2} + \left(\frac{d_1\rho}{d}\right)^2 \frac{(u_m k_m)^2 k_i k_j}{k^4}\right] \left(\frac{d \ln K}{d\rho}\right)^2 \Psi_{\rho,\rho} \tag{7}$$

(summation is performed over repeating tensor indices).

It is natural to choose the axis along the vector **u** of the mean relative velocity of the phases as one of the coordinate axes (for the sake of definiteness of the first axis); then $u_1 = u\delta_{i1}$. It can readily be seen from (7) that the coordinates of the pseudo-turbulent motion being studied are axisymmetric and in particular that $\mathbf{kDk} = D_{11}k_1^2 + D_{22}(k^2 - k_1^2)$.

Using (5) and (7) and integrating, we find expressions for the dimensionless principle self-diffusion coefficients of the particles:

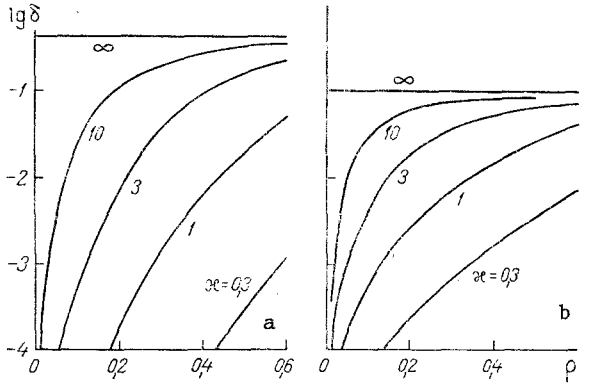

Fig. 1. Dependence of the coefficients of anisotropy of the self-diffusion of the particles (a) and fluid (b) on the volumetric concentration of the suspension at different κ (numbers next to the curves); $\rho_* = 0.6$.

$$D_1 = \frac{D_{11}}{au} = \frac{3}{D_1 - D_2}\left(\frac{2}{9\pi}\right)^{2/3}\rho^{4/3}\left(1 - \frac{\rho}{\rho_*}\right)\left(\frac{d\ln K}{d\rho}\right)^2 \times [(1-z)^2 I_0 + 2z(1-z)I_2 + z^2 I_4],$$

$$D_2 = \frac{D_{22}}{au} = \frac{3z^2}{2(D_1 - D_2)}\left(\frac{2}{9\pi}\right)^{2/3}\rho^{4/3}\left(1 - \frac{\rho}{\rho^*}\right)\left(\frac{d\ln K}{d\rho}\right)^2 \times (I_2 - I_4), \tag{8}$$

$$I_n = \int_0^1 \frac{t^n dt}{t^2 + \gamma^2}, \quad \gamma^2 = \frac{D_2}{D_1 - D_2}, \quad z = \frac{d_1\rho}{d}.$$

It is not hard to use these expressions to then obtain an equation for the coefficient γ:

$$2\gamma^2[(1-z)^2 I_0 + 2z(1-z)I_2 + z^2 I_4] = (1+\gamma^2)z^2(I_2 - I_4),$$

from which it is evident in particular that at $\kappa \to 0$ (system of gas bubbles in a liquid), $\gamma \to 0$ and $D_2/D_1 \to 0$. Similarly, at $\kappa \to \infty$ (suspension of solid particles or drops in a gas), we obtain $2\gamma^2 I_4 = (1+\gamma^2)(I_2 - I_4)$ and then $\gamma \to 0.85$ and $D_2/D_1 \to 0.42$, which corresponds to the result in [4]. The equation for γ is conveniently represented in the form

$$\frac{1-z}{z} = \frac{1-\rho}{\kappa\rho} = \frac{I_2}{I_0}\left[\left(1 + \frac{I_2 + (1-2\delta_1)I_4}{2\delta_1 I_2^2} I_0\right)^{1/2} - 1\right],$$

$$\delta_1 = \frac{\gamma^2}{1+\gamma^2}, \quad \kappa = \frac{d_1}{d_0}, \quad I_0 = \frac{1}{\gamma}\operatorname{arctg}\frac{1}{\gamma}, \tag{9}$$

$$I_2 = 1 - \gamma\operatorname{arctg}\frac{1}{\gamma}, \quad I_4 = \frac{1}{3} - \gamma^2 + \gamma^3\operatorname{arctg}\frac{1}{\gamma}.$$

Figure 1 shows the dependence of $\delta_1 = D_2/D_1$ on ρ for different κ. it is evident from the figure that this quantity is monotonic and increases very rapidly with an increase in both ρ and κ.

We can also use (8) to obtain the relations

$$D_1 = \frac{z}{(6\pi^2)^{1/6}}\frac{1+\gamma^2}{\gamma}(I_2 - I_4)^{1/2}\rho^{2/3}\left(1 - \frac{\rho}{\rho_*}\right)^{1/2}\frac{d\ln K}{d\rho}, \quad D_2 = \delta_1 D_1. \tag{10}$$

Using (5) and (7), we obtain the following expressions for the dimensionless principal self-diffusion coefficients of the fluid moles

$$E_1 = \frac{E_{11}}{au} = \sqrt{6}\left(\frac{2}{9\pi}\right)^{1/3} z\gamma\frac{I_0 - 2I_2 + I_4}{(I_2 - I_4)^{1/2}}\rho^{2/3}\left(1 - \frac{\rho}{\rho_*}\right)^{1/2}\frac{d\ln K}{d\rho}, \tag{11}$$

$$E_2 = \frac{E_{22}}{au} \equiv \frac{E_{33}}{au} = \delta_0 E_1, \quad \delta_0 = \frac{1}{2}\frac{I_2 - I_4}{I_0 - 2I_2 + I_4}.$$

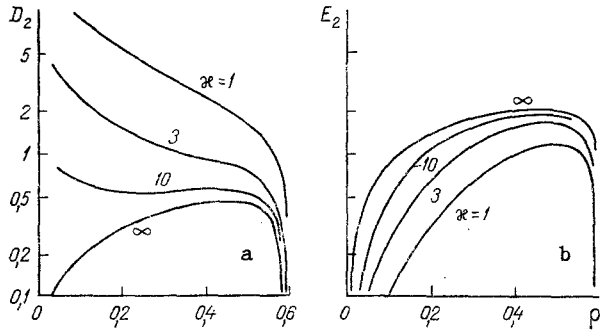

Fig. 2. Dependence of ρ of the coefficients of longitudinal self-diffusion of the particles (a) and the fluid (b) with different κ (numbers next to the curves); $\rho_* = 0.6$.

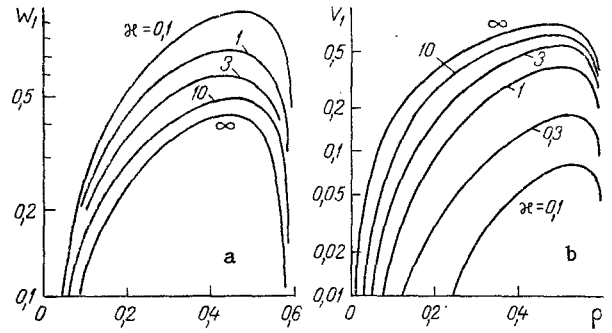

Fig. 3. Dimensionless RMS longitudinal components of the fluctuation velocities of the particles (a) and fluid (b) at $\rho_* = 0.6$ and different κ (numbers next to the curves).

For specific calculations of the self-diffusion coefficients, it is necessary to determine the function $K(\rho)$. The value $K = (1 - 5\rho/2)^{-1}$ was obtained in [5] for suspensions of moderate concentration. The same relation is very awkward, however, for highly concentrated suspensions [8]. Here, for simplicity we will use the approximate formula

$$K = \frac{1}{(1-\rho)^{5/2}}, \quad \frac{d\ln K}{d\rho} = \frac{5/2}{1-\rho}, \tag{12}$$

This formula is valid as an approximation throughout the interval $0 \leq \rho \leq \rho_*$. The relations D_1 and E_1 from (10) and (11) corresponding to (12) are shown in Fig. 2.

Now let us calculate the standard deviations of the velocities of the particles and fluid moles. After integrating with the use of Eqs. (7) for the spectral densities, we obtain expressions for the dimensionless RMS components of particle fluctuation velocity:

$$W_1 = \frac{\langle w_1'^2 \rangle^{1/2}}{u} = \left(1 - \frac{4}{3}z + \frac{8}{15}z^2\right)^{1/2} \rho \left(1 - \frac{\rho}{\rho^*}\right)^{1/2} \frac{d\ln K}{d\rho},$$
$$W_2 = \frac{\langle w_2'^2 \rangle^{1/2}}{u} \equiv \frac{\langle w_3'^2 \rangle^{1/2}}{u} = \frac{z}{\sqrt{15}} \rho \left(1 - \frac{\rho}{\rho^*}\right)^{1/2} \frac{d\ln K}{d\rho} \tag{13}$$

and fluid fluctuation velocity:

$$V_1 = \frac{\langle v_1'^2 \rangle^{1/2}}{u} = \sqrt{\frac{8}{15}} z\rho \left(1 - \frac{\rho}{\rho^*}\right)^{1/2} \frac{d\ln K}{d\rho}, \tag{14}$$
$$V_2 = \frac{\langle v_2'^2 \rangle^{1/2}}{u} \equiv \frac{\langle v_3'^2 \rangle^{1/2}}{u} \frac{z}{\sqrt{15}} \rho \left(1 - \frac{\rho}{\rho_*}\right)^{1/2} \frac{d\ln K}{d\rho}.$$

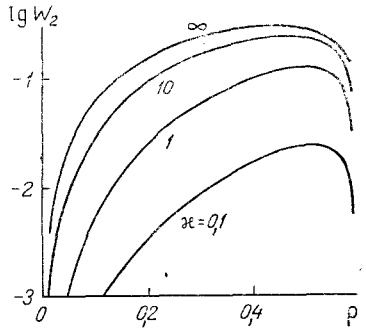

Fig. 4. Dimensionless transverse RMS components of fluctuation velocities; the notation is the same as in Fig. 3.

Figures 3 and 4 show the relations from the quantities W_1, V_1, and $W_2 = V_2$ from (13) and (14) in accordance with Eqs. (12). The general character of these relations is similar to the character of the relations obtained in [3, 4]. They all have maxima at values of ρ close to ρ_*.

The ratio V_2/V_1 does not depend on ρ or κ, while the ratio W_2/W_1 rapidly increases with an increase in both of these parameters. Thus, the anisotropy of the pseudo-turbulent particle pulsations decreases with an increase in the concentration of the suspension and the ratio of the densities of the particle material and fluid. The anisotropy of the fluid pulsations remains unchanged in this case.

Equations (7) also make it possible to express the spectral densities $\Psi_{\rho,\rho}(\omega, k)$ through $\Psi_{v_i,w_j}(\omega, k)$ from (5) and to then find means of the form $\langle v_i'\omega_j'\rangle$; only the means with i = j turn out to be nontrivial. These representations for spectral density can subsequently be used to calculate the corresponding correlation functions, which is useful in determining the characteristic scales of pseudo-turbulent motion. Omitting such calculations here, we note only that the linear scale of pulsations has the order k_0^{-1}, as might be expected. The time scale has the order uk_0. From this, it is in principle possible to evaluate the limits of applicability of the above-used noninertial approximation. In fact, the condition of triviality of the inertial terms in (1) can be written in the form $d_m uk_0 \ll \beta K$. Meanwhile, $u \approx (d_1 - d)g_u/\beta K$, where d_m is the density of the material of the heavier phase and g_u is the component of g in the direction of $\mathbf{u}$. Assuming in accordance with (5) that $k_0 \approx a^{-1}$, we have the inequality $a^3 \lesssim K^2\nu_0^2 d_0^2[d_m(d_1 - d_0)g_u]^{-1}$. At $g_u \sim 10^3$ cm/sec^2 for moderately concentrated liquid suspensions ($\nu_0 \sim 10^{-2}$ cm^2/sec) or aerosols ($\nu_0 \sim 10^{-1}$ cm^2/sec), we obtain $a \lesssim 10^{-2}$ cm. If the particle size does not correspond to this estimate, then the theory must be generalized in two directions: On the one hand, it is necessary to consider the inertia of the particles (and the apparent additional mass, if the particles are suspended in a fluid in drop form); on the other hand, it is necessary to use representations for the phase interaction force which are nonlinear with respect to the relative velocity of the phases. Such a generalization was made in [4] for aerosols; here, satisfactory agreement was obtained with experimental data from the determination of the self-diffusion coefficient in the transverse direction in a fluidized bed.

Along with the above restrictions on the maximum particle size for which the theory is valid, there is another restriction on the minimum particle size. The latter is due to the fact the particles are brought not only into pseudo-turbulent motion, but also normal Brownian motion. Brownian motion was not considered in the above case. It is clear that this omission will be acceptable only if the characteristic velocity or energy associated with the Brownian motion of the particles is much lower than their pseudo-turbulent velocity or energy corresponding to the degree of freedom in question, i.e., it is necessary to satisfy the inequality $kT \ll m\langle w_i'^2\rangle$, where kT is the temperature in energy units. At room temperature, this inequality leads to estimates $a \gtrsim 10^{-4}$-10^{-3} cm and $a \gtrsim 10^{-5}$-10^{-4} cm of the approximate validity of the proposed model for evaluation of the transverse and longitudinal components of fluctuation velocity, respectively. In principle, the effect of Brownian motion can be allowed for by introducing into (1) and (2) a new random force that will be isotropic, be statistically independent of concentration fluctuations, and have the spectral density of white noise.

NOTATION

a, particle radius; d, density; D_{ij}, E_{ij}, self-diffusion coefficients of the particles and fluid moles; D_i, E_i, dimensionless principal self-diffusion coefficients; F, force acting

on a particle; g, acceleration due to gravity; k, wave vector; k_0, maximum value of k determined in (5); K, function describing the effect of physical constraint on the viscous force of the phase interaction; m, mass of particle; n, numerical concentration of particles; p, pressure; u, mean relative velocity of the phases; v, w, velocities of the fluid and particles; V_i, W_i, dimensionless fluctuation velocities; $\delta_0 = E_2/E_1$; $\kappa = d_1/d_0$; μ, ν, absolute and kinematic viscosities; ρ, ρ_*, volumetric concentration of particles and the concentration corresponding to the packing density; ω, frequency; Φ, Ψ, spectral densities. The subscripts 0 and 1 respectively denote the fluid and particles; the primes denote fluctuation quantities; the brackets denote averaging.

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NUMERICAL MODELING OF TURBULENT FIELDS OF VELOCITY, TEMPERATURE, AND CONCENTRATION IN A RECTANGULAR CHANNEL

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UDC 536.24:532.54

The finite elements method is used to solve a system of three-dimensional transport equations in a rectangular channel.

Introduction. The numerical modeling of flow and heat and mass transfer in complicated channels is a new and important means of investigation. The popularity of numerical experiments employing computers stems from many factors, the most important being the completeness of the information, the speed with which it is obtained, and the possibility of modeling a wide range of situations – including some that cannot be realized in a physical experiment.

The mathematical model and application package employed in the present article can be used with success to describe and model a wide range of so-called parabolic flows in closed channels of complex form. At the current stage of investigation, we will restrict ourselves to flows without buoyancy and we will consider only the longitudinal component of the velocity vector. The model contains two assumptions connected with Newtonian fluids. The system of transport equations is based on the equations of continuity, motion, energy, diffusion, and heat conduction. The numerical realization was accomplished on the basis of the finite elements method in the Galerkin modification. Below, we report the details of the formulation and method of solution of the problem. We also generalize the results and present certain other results of numerical experiments.

Mathematical Model. Initial Equations. To describe the hydrodynamic part of the problem, we will use the model of parabolic flow in a closed rectangular channel (Fig. 1). the Navier–Stokes equations for the longitudinal component of the velocity vector have the form:

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